(1)

D1(H)-Maths, paper-1st, Gr-A, summation of series C+iS method ...

4 W.O $1-\dfrac{\cos 2\theta}{\lfloor 2}+\dfrac{\cos 4\theta}{\lfloor 4}-\dfrac{\cos 6\theta}{\lfloor 6}+\dots$

Let $C=1-\dfrac{\cos 2\theta}{\lfloor 2}+\dfrac{\cos 4\theta}{\lfloor 4}-\dfrac{\cos 6\theta}{\lfloor 6}+\dots$

and $S=1-\dfrac{\sin 2\theta}{\lfloor 2}+\dfrac{\sin 4\theta}{\lfloor 4}-\dfrac{\sin 6\theta}{\lfloor 6}+\dots$

$$\therefore C+iS=1-\frac{1}{\lfloor 2}(\cos 2\theta+i\sin 2\theta)+\frac{1}{\lfloor 4}(\cos 4\theta+i\sin 4\theta)-\frac{1}{\lfloor 6}(\cos 6\theta+i\sin 6\theta)+\dots$$

$$=1-\frac{1}{\lfloor 2}e^{2i\theta}+\frac{1}{\lfloor 4}e^{4i\theta}-\dots \quad \text{let } e^{i\theta}=x$$

$$=1-\frac{x^2}{\lfloor 2}+\frac{x^4}{\lfloor 4}-\dots$$

$$=\cos x=\cos(e^{i\theta})=\cos(\cos\theta+i\sin\theta)$$

$$=\cos(\cos\theta)\cdot\cos(i\sin\theta)-\sin(\cos\theta)\sin(i\sin\theta)$$

$$=\cos(\cos\theta)\cos h(\sin\theta)-i\sin(\cos\theta)\sinh(\sin\theta)$$

$$\therefore C=\cos(\cos\theta)\cos h(\sin\theta)$$

6 W.O $\dfrac{1}{2}\cos\alpha\sin\alpha+\dfrac{1\cdot 3}{2\cdot 4}\cos^2\alpha\sin 2\alpha+\dfrac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\cos^3\alpha\sin 3\alpha+\dots$

Let $S=\dfrac{1}{2}\cos\alpha\sin\alpha+\dfrac{1\cdot 3}{2\cdot 4}\cos^2\alpha\sin 2\alpha+\dfrac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\cos^3\alpha\sin 3\alpha+\dots$

and $C=\dfrac{1}{2}\cos\alpha\cos\alpha+\dfrac{1\cdot 3}{2\cdot 4}\cos^2\alpha\cos 2\alpha+\dfrac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\cos^3\alpha\cos 3\alpha+\dots$

$$\therefore C+iS=\frac{1}{2}\cos\alpha(\cos\alpha+i\sin\alpha)+\frac{1\cdot 3}{2\cdot 4}\cos^2\alpha(\cos 2\alpha+i\sin 2\alpha)+\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\cos^3\alpha(\cos 3\alpha+i\sin 3\alpha)+\dots$$

$$=\frac{1}{2}\cos\alpha\, e^{i\alpha}+\frac{1\cdot 3}{2\cdot 4}\cos^2\alpha\, e^{2i\alpha}+\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\cos^3\alpha\, e^{3i\alpha}+\dots$$

Let $x=\cos\alpha\cdot e^{i\alpha}$

$$\therefore C+iS+1=1+\frac{1}{2}x+\frac{1\cdot 3}{2\cdot 4}x^2+\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}x^3+\dots$$

$$=1+\frac{1}{2}x+\frac{\frac{1}{2}\cdot\frac{3}{2}}{1\cdot 2}(x)^2+\frac{\frac{1}{2}\cdot\frac{3}{2}\cdot\frac{5}{2}}{1\cdot 2\cdot 3}x^3+\dots$$

②

$$= (1-x)^{-1/2} = \frac{1}{(1-x)^{1/2}} = \frac{1}{\sqrt{1-\cos\alpha\cdot e^{i\alpha}}}$$

$$= \frac{1}{\sqrt{1-\cos\alpha(\cos\alpha+i\sin\alpha)}} = \frac{1}{\sqrt{1-\cos^2\alpha - i\sin\alpha\cos\alpha}}$$

$$= \frac{1}{\sqrt{\sin^2\alpha - i\sin\alpha\cos\alpha}}$$

$$\text{or } c+is+1 = \frac{1}{\sqrt{\sin\alpha\{\sin\alpha - i\cos\alpha\}}}$$

$$= \frac{1}{\sqrt{\sin\alpha}} \times \frac{1}{\sqrt{\cos(\pi/2-\alpha) - i\sin(\pi/2-\alpha)}}$$

$$= \frac{1}{\sqrt{\sin\alpha}}\{\cos(\pi/2-\alpha) - i\sin(\pi/2-\alpha)\}^{-1/2}$$

$$= \frac{1}{\sqrt{\sin\alpha}}\{\cos(\pi/4-\tfrac{\alpha}{2}) + i\sin(\pi/4-\alpha/2)\}$$

$$\therefore c+1 = \frac{1}{\sqrt{\sin\alpha}}\cos(\pi/4-\alpha/2)$$

$$\text{and } s = \frac{1}{\sqrt{\sin\alpha}}\{\sin(\pi/4-\alpha/2)\}$$

$$= \frac{1}{\sqrt{\sin\alpha}}\{\sin\pi/4\cos\alpha/2 - \cos\pi/4\sin\alpha/2\}$$

$$= \frac{1}{\sqrt{\sin\alpha}}\left\{\frac{1}{\sqrt{2}}\cos\frac{\alpha}{2} - \frac{1}{\sqrt{2}}\sin\frac{\alpha}{2}\right\}$$

$$= \frac{1}{\sqrt{\sin\alpha}} \times \frac{1}{\sqrt{2}}\{\cos\alpha/2 - \sin\alpha/2\}$$

$$= \frac{1}{\sqrt{2\sin\alpha}}(\cos\alpha/2 - \sin\alpha/2) \quad \Omega$$